A COMPARISON BETWEEN OBSERVED AND GEOSTROPHIC WINDS IN LOW LATITUDES

by

Charles L. Jordan
University of Chicago

Summary - A latitudinal profile of the mean geostrophic deviation is computed from surface and low level wind data and surface pressure gradients. The resulting geostrophic deviation is discussed in terms of the equations of motion.

In the meteorological literature the question arises perpetually to what extent the geostrophic wind speed is a valid approximation to the true wind speed in low latitudes. This study attempts to establish the mean magnitude of the departure of the actual wind speed from the geostrophic by considering low level data over the oceans.

Direct comparison of the wind at the top of the layer of surface frictional influence with the geostrophic wind speed at that level was not possible. Instead, this study has been carried out by (a) comparing the 2000 ft wind speed at low latitude stations with the geostrophic indicated by the sea level isobars and (b) by comparing the geostrophic speed with a "corrected" wind speed obtained by adding to the surface wind a factor to compensate for friction.

Only the total wind speed has been used in the quantitative treatment of the data. However, in reasoning from the results certain conclusions are reached in regard to the relation between the actual and geostrophic wind directions. Observational studies have indicated that the angular turning near the surface is quite small over the oceans whenever the lapse rate is steep. Riehl and collab. [1] found the mean angular turning in the northeast trades of the East Pacific to be less than 5° in the lowest 5000 ft. Further, Gordon [2] found the mean angular turning between the surface and 2000 ft in the North Atlantic, 60° to 60°N, to be less than 8° whenever the lapse rate was near the dry adiabatic. In view of the nearly dry adiabatic lapse rates near the surface over the tropical and subtropical oceans in the mean, near zero horizontal temperature gradients and small changes of wind direction with height appear quite reasonable.

Comparison of 2000 ft winds with the geostrophic

Wind data from four low latitude island stations have been used to compute the deviation of the 2000 ft speed from the geostrophic. These wind data and the surface isobaric spacing have been taken from the Northern Hemisphere Synoptic Weather Maps [3] for the period September 1949 through January 1950. Admittedly, the surface isobars cannot be drawn over the oceans with much accuracy. However, averages over a large number of cases should tend to eliminate the effect of errors of this type. No selection was made on the type of surface isobaric curvature but due to the location of the stations slight anticyclonic curvature prevailed in the most cases. Thus correction for curvature would have resulted in a gradient wind slightly greater than the geostrophic.

The mean 03Z wind speed at the 2000 ft level (c_{2000}), the mean geostrophic speed (c_g), and the mean geostrophic departure ($c_g - c_{2000}$) are shown for the four stations in the table below.[1]

Station	Lat.	No. of reports	c_{2000} (mps)	c_g (mps)	$c_g - c_{2000}$
Bermuda	32°N	55	8.4	9.6	1.2
Wake	19°N	72	8.1	10.6	2.5
Johnston	17°N	88	10.0	14.2	4.2
Guam	13°N	61	7.9	13.1	5.2

These data indicate that the geostrophic departure increases equatorward and at the lower latitudes reaches a value quite large in comparison with the mean wind.

This computed departure can be identified with the true geostrophic departure only if it is assumed that the pressure gradient remains unchanged between the surface and 2000 ft. The validity of this assumption has been checked by computing from the geostrophic thermal wind equation, the increase or decrease of the zonal wind between the surface and 2000 ft which can be attributed to the mean latitudinal temperature gradient. For this calculation the thermal wind equation has been written

$$\Delta u = \left[-\frac{u_o}{T_o}\frac{\partial T}{\partial z} + \frac{g}{f\bar{T}}\frac{\partial \bar{T}}{\partial y}\right]\Delta z$$

where Δu is the increment to be applied to u_o, the zonal component of the mean surface wind; T_o is the surface temperature; $\bar{T}$ is the mean temperature between the surface and 2000 ft; f is the Coriolis parameter and g is the acceleration of gravity. Δz has been taken as 2000 ft and $\frac{\partial T}{\partial z}$ as the dry adiabatic rate since this calculation is made in the mixed "subcloud" layer [4]. The mean latitudinal temperature gradients over the Atlantic and Pacific Oceans were determined for September and November from the Atlas of Climatic Charts of the Oceans [5]. The decrease of the zonal component between the surface and 2000 ft in mps which can be attributed to the thermal wind is shown in the following table:

Lat.	September	November
10°N	-0.4	0
15°N	0.1	0.3
20°N	0.4	1.2
25°N	0.4	1.1
30°N	0.5	1.2

These values cannot be taken as strictly correct since the application of dynamic equations to mean data is always open to question. However, the order of magnitude can hardly be doubted. The geostrophic departure shown by the table above increases south of 20°N, whereas the thermal contribution approaches zero at these latitudes. Only in the latitudes north of 20°N where the geostrophic departure is quite small is this computed thermal contribution of the same order of magnitude as the geostrophic departure. Thus the change of pressure gradient

[1] These results should not be affected appreciably by a diurnal variation in the low level wind speed since compensating changes in the pressure gradient would be expected.

through the frictional layer due to the mean latitudinal temperature gradient cannot be considered of primary significance in explaining any deviation from geostrophic balance in low latitudes.

Comparison of "corrected" surface winds with the geostrophic

Wind data are available from only a limited number of low latitude stations. At many of these, local wind systems due to orography and surface heating may be such as to distort the low level flow. Consequently, this study has been extended by comparing "corrected" surface winds with the geostrophic.

The mean increase of the wind speed from the surface to the top of the friction layer computed by Riehl and collab. [1] from several months of weather ship pibal observations in the trade wind belt of the east Pacific Ocean was found to be, at most, 40 per cent of the surface wind. This is in agreement with a study by Gordon [6] for the North Atlantic. Mean ratios of the surface and 1 km wind computed from wind records of several low latitude ship stations for the fall and winter of 1945-46 are shown in the table below.

Ship Stations	Number of Observations	Mean Ratio 1 km to sfc wind
27°N - 125°W	43	1.1
26°N - 149°W	30	1.3
22°N - 134°E	115	1.2
18°N - 165°E	29	1.2
17°N - 130°E	30	1.1
1 °N - 154°E	41	1.2

In addition to indicating ratios less than 1.4, in agreement with Riehl and Collab. and Gordon, these data show that any latitudinal variation in the ratio must be rather insignificant. The near dry adiabatic low level lapse rates which are in general present over the tropical and subtropical oceans contribute to the constancy of this ratio.

On the basis of the evidence presented above it appears that over the oceans multiplication of the surface wind speed by 1.4 should give a value in the mean larger than the mean observed wind in the region of 2000-3000 ft.[2] Thus the comparison of geostrophic and observed winds was extended by using a corrected wind (c_c), which was obtained by multiplying the surface wind by 1.4, and the geostrophic wind (c_g) which was determined from the sea level isobars. Only at those times when the pressure gradient was well-defined could the geostrophic wind be determined. Consequently, the wind values throughout this study are some-above the average for the respective latitudes.

The statistical sample for this study was obtained from the Northern Hemisphere Synoptic Weather Maps [3], [7]. A sample of about 1400 observations was taken from the Atlantic and Pacific Oceans between 7°N and 38°N from the periods October 1945 to January 1946 and January to April 1949. The reports were from moving ships, stationary ship stations, and small atolls,

---

[2]Since conditions over the low latitude oceans are quasi-barotropic in a rather deep layer, mean speeds in the region 2000-4000 ft can be considered essentially those at the top of the friction layer, the mean depth of which is not known.

such as Johnston and Kwajalein. Reports from larger island groups such as the Philippines and the Antilles were not used. The wind reports used were located in well-defined pressure patterns with slight anticyclonic or neglible curvature.

For each wind report c_o, c_g and $c_g - c_o$ were determined. These were grouped in belts of 4° latitude width and mean values were calculated. The principal results are shown by figures 1 and 2. Since the 1.4 factor in general gives a corrected wind greater than the actual and since the anti-cyclonic curvature has not been corrected for, the computed geostrophic departure, $c_g - c_o$, should represent a minimum value of the actual departure. The computed geostrophic departure approaches zero near 30°N (Fig. 2) and increases both northward and southward. South of 30°N the increase follows a smooth curve and reaches a departure of about 10 mps near 10°N. The increase of thermal stability may account for the increase of the geostrophic departure north of 30°N (cf. [4]).

The geostrophic departure expressed as a percentage of the corrected wind is shown in figure 3. Since the corrected wind is near 10 mps at all latitudes, the shape of figure 3 is not essentially different from figure 2. South of 15°N the departure is greater than 50 per cent of c_o and south of 10°N the departure exceeds c_o by more than 100 per cent.

The table below shows that the departures read from the curve of figure 2 agree closely with those computed for the four stations in the previous section.

Station	$c_g - c_{2000}$	Lat.	$c_g - c_o$ (from Fig. 2)
Bermuda	1.2 mps	32°N	1.0 mps
Wake	2.5	19°N	2.4
Johnston	4.2	17°N	3.8
Guam	5.2	13°N	6.0

Both sets of data presented above indicate that the deviation of the actual wind speed from the geostrophic becomes progressively larger with decreasing latitude. Further, the close agreement between these independent calculations leads to greater confidence in the accuracy of figure 2.

It might appear that the very large departures south of 15°N could be biased by the fact that analysis errors which tended to crowd the isobars would be more likely than those in the opposite sense. This would appear particularly likely since the pressure gradients are weak and since this latitude zone is near the equatorward limit of the analysis. However, the following check did not reveal any bias in this sense. The mean geostrophic departure was computed from Northern Hemisphere Synoptic Maps [7] as above, for June and July 1946 in the region of the Marshall Islands. During this period the low latitude data in that area were greatly increased, due to the atomic bomb tests at Bikini, and the pressure analysis should have been more reliable. The mean geostrophic departure computed from 50 reports during these two months at 11-12°N was 7.8 mps, compared to about 7.5 mps from figure 2.

Figure 4 shows the percentage of reports in each latitude belt in which the geostrophic was greater than the corrected wind. Practically all reports in the lowest latitudes show $c_g > c_o$ while near 30°N, c_o is nearly equally distributed above and below c_g. The computed departure cannot be applied to individual wind reports with any degree of confidence since at any time the change of wind speed through the friction layer may deviate appreciably from the 40 per cent increase used in this study and since the geostrophic wind usually cannot be determined with any degree of accuracy. However, figure 4 shows that in over 90 per cent of the cases south of 15°N latitude it should be expected that the geostrophic wind would be an overestimate and that in the mean the deviation of the geostrophic from the actual would be at least as large as that shown by figure 2.

Implications

It is logical to question whether the large geostrophic departures shown by figure 2 are physically realistic and can be interpreted in terms of the horizontal equations of motion. The equations of motion in a natural coordinate system s, n can be written:

$$(1) \quad \frac{dc}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial s} - F$$

$$(2) \quad 0 = -\frac{1}{\rho}\frac{\partial p}{\partial n} - fc + \frac{c^2}{R}$$

where s and n are oriented along and normal to the streamlines, respectively, as shown in figure 5. F is the surface frictional force assumed to be oppositely directed to the wind velocity c, ρ is density, f is the Coriolis parameter, p is pressure and R is the radius of curvature of the trajectories. R is taken positive for anticyclonic curvature.

It will be assumed that mean circulation patterns in the northeast trades, such as shown in figure 5, are sufficiently steady state so that the above equations can be applied for order of magnitude calculations. First, if we consider the balance of forces along the streamlines using (1), this equation can be written:

$$(3) \quad c\frac{\partial c}{\partial s} = -\frac{1}{\rho}\frac{\partial p}{\partial s} - F$$

since $\frac{\partial c}{\partial t} = 0$ in the mean pattern. Further, at points of maximum and minimum speed, for example point A figure 5, $\frac{\partial c}{\partial s} = 0$ and (3) can be written:

$$(4) \quad -\frac{1}{\rho}\frac{\partial p}{\partial s} - F = 0 \quad \text{or} \quad f c_{gn} - F = 0$$

where c_{gn} is the component of the geostrophic wind normal to the stream-lines. Clearly, at the points where (4) is satisfied a balance of forces can be effected only when the streamlines have a component toward lower pressure. Equatorward from the maximum $\frac{\partial c}{\partial s} < 0$ and the magnitude of $f c_{gn}$ decreases provided the magnitude of F remains constant or decreases. However, the component of geostrophic wind normal to the streamlines may increase equatorward provided the decrease in the magnitude of $c\frac{\partial c}{\partial s} - F$ is over-compensated for by the decrease of f.

If we now consider the forces acting normal to the streamlines in the anticylonic case, such as shown in figure 5, (2) may be written:

$$(5) \quad f(c_{gs} - c) + \frac{c^2}{R} = 0$$

where c_{gs} is the component of the geostrophic wind along the streamlines. It should be noted that a balance of forces is not possible in the case of anticylonic curvature if $c_{gs} > c$. In the balanced case $c_{gs} = c_g \cos\alpha$ where α is the angle between streamlines and isobars. Thus the angle of cross isobaric flow can be computed if c_g, c and the curvature are known. If we choose the case of straight flow, $R = \infty$, then

$$(6) \quad \alpha = \cos^{-1}\left[\frac{c}{c_g}\right]$$

Using values of c_g and c from figure 2, α and c_{gn} have been computed for several latitudes:

Latitude	α	c_{gn}
25°N	23°	3.9 mps
20°N	38°	5.8
17.5°N	45°	6.7
15°N	48°	7.2
10°N	61°	6.3

From the reasoning carried out with the balance of forces along the stream-lines, this cross isobaric flow is necessarily directed toward lower pressure. Addition of a curvature term to (6) would increase the angle of outflow in the case of anticyclonic curvature and decrease it for cyclonic curvature.

In the discussion above the angular turning of the wind with height has been neglected. Presumably, vertical mixing should contribute to a frictional term with a component normal to the wind direction at the level in question whenever a systematic turning with height is present. If the wind veers with height, addition of a normal frictional term to (5) would permit a smaller angle of outflow for a given geostrophic departure. However, as discussed above, observational studies [1] [2] have shown the angular turning in the "subcloud" layer to be quite small. Consequently, the angle of outflow would not be substantially changed by introducing a normal frictional term.

Admittedly, the calculations carried out in this section have been very rough but they have shown that mean geostrophic departures of the magnitude shown by figure 2 can be accounted for in the simplified equations of motion only by introducing large cross isobaric flow. The mean streamline and isobaric patterns do intersect at rather large angles over the tropical and subtropical oceans. For instance, the mean streamlines and isobars for the East Pacific (Fig. 6) show that the angle of outflow in many cases is as large as shown in the table above. Perhaps of more significance than showing the magnitude of the angle of outflow, these calculations have indicated that the difficulties of using and interpreting the geostrophic wind equation south of about 20°N become very great.

Acknowledgment

The writer wishes to thank Professors H. Riehl and E. Palmen for their assistance during various phases of the research and preparation of this report.

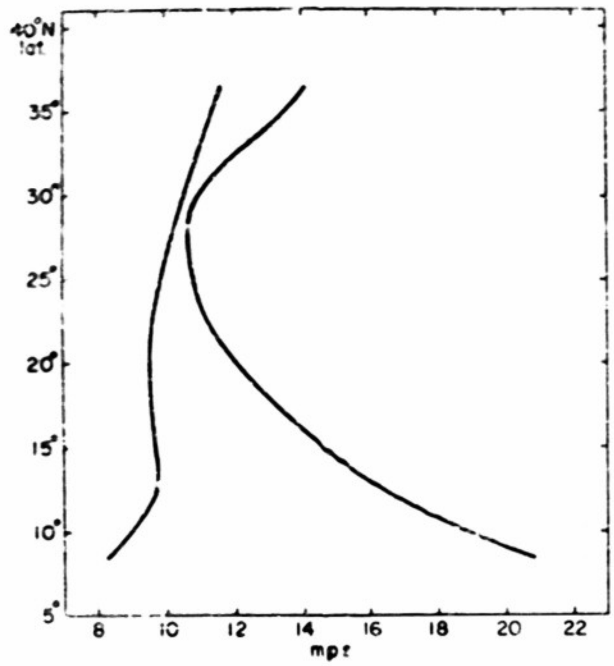

Fig. 1. Latitudinal profiles of the mean "corrected" surface wind (left curve) and mean geostrophic wind (right curve).

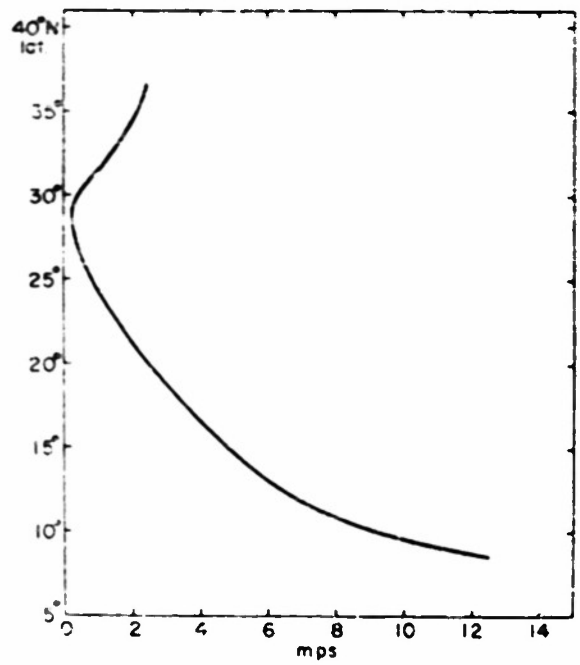

Fig. 2. Latitudinal profile of the mean geostrophic departure representing the difference between the two curves of figure 1.

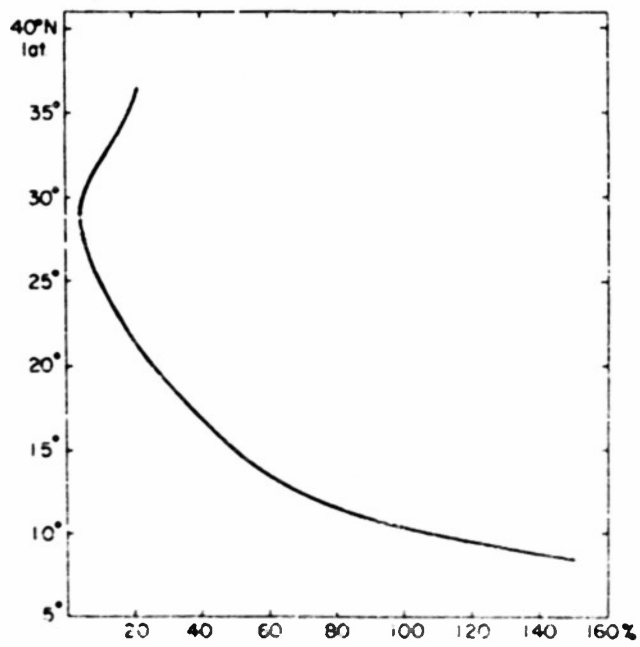

Fig. 3. Profile of the mean geostrophic departure expressed in percentage of the "corrected" wind.

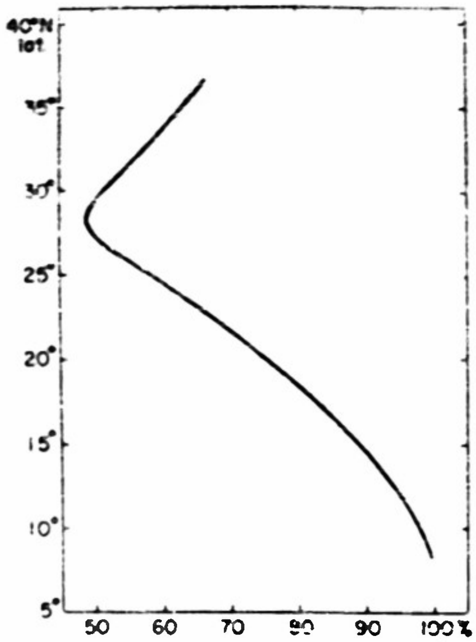

Fig. 4. Percentage of individual reports showing the geostrophic wind greater than the "corrected" wind as a function of latitude.

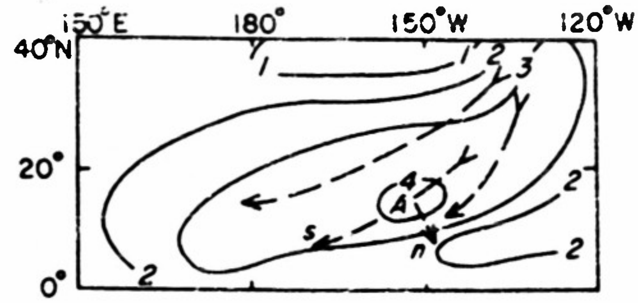

Fig. 5. Mean wind chart for a portion of the East Pacific Ocean for July. Solid lines are isotachs in Beaufort force, dashed lines are streamlines (Adapted from 5).

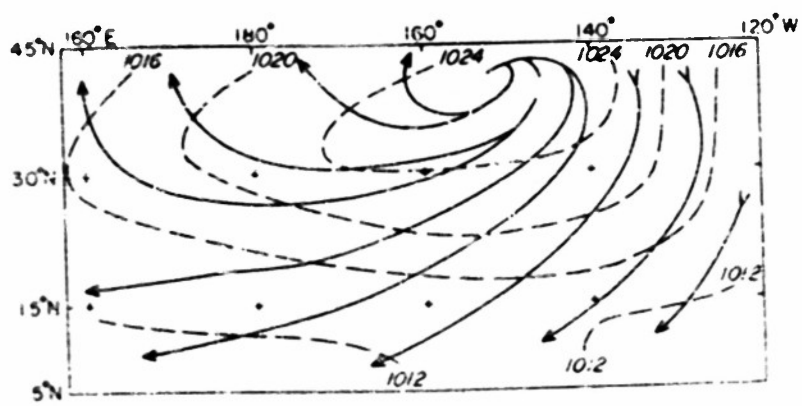

Fig. 6. Mean streamlines (solid) and mean isobars (dashed) for a portion of the East Pacific Ocean for July.